AMERICAN UNIVERSITY OF BEIRUT

MIDTERM EXAM.: STAT 230 Summer , 2008

Name: **Section:**

I.D. :

For corrector only

Grade of Written Part	Grade of Multiple Choice	Midterm Exam.. Grade
1 21 2. 15 3 16	Number of Correct:.......x 4=....48....	100

PART ONE : MULTIPLE CHOICE(No Penalty). Circle your answer. Use E when the answer is not any of the given options. For each question answered correctly you get 4 points.

Let X be a continuous random variable of p.d.f. : $f(x)=\begin{cases} \frac{x}{2}, & for\ 0 \le x \le 1 \\ \frac{1}{2}, & for\ 1 \le x \le 2.5 \end{cases}$

Then, P(0.5<X<2)=...........

A. 21/32 **B. 11/16** **C. 9/16** **D. 17/32** **E.**

Let A and B be independent events of the sample space S. Assume P(B)=x and P[(A ∪ B)']=y. Then, P(A) in terms of x and y will be:......

A. (1-x-y)/(1-x) **B. (x+1)/(x+y)** **C. (x+y+1)/(x+y+2)** **D. (1-x-y) / (1+x+y)** **E.**

A wine-fan randomly bought 4 bottles of wine from a store having 8 bottles that are respectively, one year old, two years old, ... , eight years old. [redacted]
Find the probability that the youngest bottle selected was 3 years old.

A. 2/9 **B. 1/5** **C. 3/8** **D. 1/7** **E.**

The value of the sum S= $\sum_{x=0}^{\infty} \frac{x^2 e^{-4} 4^x}{x!}$ is:........

A. 18 **B. 20** **C. 42** **D. 32** **E.**

$Let\ P(A)=0.43\ ;\ P(B)=0.35\ and\ P(A' \cap B')=0.40\ .\ Then, P(A \cap B)=.......$

A. 0.18 **B. 0.20** **C. 0.28** **D. 0.23** **E.**

A woman is to distribute the twelve identical cookies, she has prepared, to her 5 children of different ages. In how ways can she do the job if the youngest is to get at least 2 pieces and the others are to get at least one piece each?How many options does this woman have? **(The concern here is how many each child has).**

A. 244 B. 540 C. 210 D. 720 E.

A student is a fan of breakfast cereals. He has stored five kinds of corn flakes and three kinds of rice krispies. Every morning he takes a large bowl containing warm milk(which is a must),to which he adds:

3 kinds of corn flakes(one spoon from each),

from 0 to 3 kinds of rice krispies(0 or 1 spoon from each kind)

and from 0 to 4 spoonfuls of sugar(to provide a required sweetness).

How many different flavors can he come up with?

A. 2400 B. 2560 C. 780 D. 400 E.

Five cards are randomly drawn from a complete deck of cards. Find the probability that there is at least one diamond, assuming that drawing is made with replacement.

A. 781/1024 B. 57/512 C. 65/512 D. 823/1024 E.

Five cards are randomly drawn from a complete deck of cards. Find the probability that there is at least one diamond, assuming that drawing is made without replacement.(Answer is rounded)

A. 0.8875 B. 0.5578 C. 0.7785 D. 0.6565 E.

A box contains ten packs of playing cards. Two of the packs are complete decks of 52 cards,

three packs that do not contain any spades,

and five packs that do not contain any hearts.

One of the ten packs is randomly selected, and one card is randomly removed from it and found to be red. What is the probability that it is coming from a complete pack?

A. 4/13 B. 3/14 C. 2/23 D. 1/3 E.

A continuous random variable ,X , has a p.d.f. : $f(x)=(\frac{4x^3}{k^4})e^{-\left(\frac{x}{k}\right)^4} : 0<x<\infty.$

If the median is $\pi_{0.5}=5,$ then $k=$........

A. $2(\ln 5)^{1/4}$ B. $20(\ln 2)$ C. $5/(\ln 2)^{1/4}$ D. $2/\ln 5$ E.

A random variable, X, has the distribution function : $F(x)=\begin{cases} 0, & x\le 0 \\ \frac{x}{8}, & 0<x<2 \\ \frac{x^2}{16}, & 2\le x<4 \\ 1, & x\ge 4 \end{cases}$

Then the mean of X is............

A. 49/12 B. 37/12 C. 19/12 D. 31/12 E.

PART II: WRITTEN QUESTIONS:(You must write adequate explanation for each major step).

I. ***(6+3+3+3+3+3+3=21points)*** One four sided and one six sided dice are tossed together, and the sum of the outcomes, X, is observed.(Answer the following parts)

a) Form a probability table for X and calculate the probability that the sum is smaller than 4.

X	2	3	4	5	6	7	8	9	10
f(x)=P(X=x)	1/24	2/24	3/24	4/24	4/24	4/24	3/24	2/24	1/24

$$P(X<4) = P(X=2) + P(X=3) = \frac{3}{24}$$

6

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10

b) If the dice are thrown over and over, find the probability that a sum of 4 appears before a sum of 7.

$$P(x=4 \mid x=4 \cup x=7) = \frac{P(x=4)}{P(x=4 \cup x=7)} = \frac{P(x=4)}{P(x=4)+P(x=7)} = \frac{3}{7}$$

3

c) If the dice are tossed 12 times, what is the probability that a sum of seven will appear twice?

Binomial; $n=12,\ p=\frac{4}{24},\ x=2$

$$P(x=2) = f(2) = \binom{12}{2}\left(\frac{4}{24}\right)^2\left(1-\frac{4}{24}\right)^{12-2} = 0.2961$$

3

d) If the dice are tossed over and over, what is the prob. that the tenth trial will result in the third sum of 7?

Negative Binomial; $p=\frac{4}{24},\ r=3,\ x=10$

$$P(x=10) = f(10) = \binom{9}{2}\left(\frac{4}{24}\right)^3\left(1-\frac{4}{24}\right)^7 = 0.0465$$

3

e) Let Y count the number of trials needed to get the fourth sum of 7. What is the moment generating function, M(t) of Y? Write the formula (do not derive) in terms of just t.

Negative binomial: $p = \frac{4}{24}$, $r = 4$

$$M(t) = \frac{\left[\frac{4}{24} e^t\right]^4}{\left[1 - \frac{20}{24} e^t\right]^4}$$

3

f) Find the mean of Y using the formula written in part (e) above.

$$M'(t) = 4\left[\frac{\frac{4}{24} e^t}{1 - \frac{20}{24} e^t}\right]^3 \left(\frac{(1-\frac{20}{24}e^t)(\frac{4}{24}e^t) - (\frac{4}{24}e^t)(-\frac{20}{24}e^t)}{(1-\frac{20}{24}e^t)^2}\right)$$

$$M'(0) = 4 \cdot \left(\frac{4/24}{4/24}\right)^3 \left(\frac{(1-\frac{20}{24})(\frac{4}{24}) + (\frac{4}{24})(\frac{20}{24})}{(1-\frac{20}{24})^2}\right) = 24$$

3

II. ***(5+5+5=15points)*** A box contains seven balls, one marked WIN(W) and six marked LOSE(L). Three players A, B, and C take turns selecting a ball from the box, one at a time (A starts, then B, then C, then A,........). The first player to select the W is the winner. (Answer the following parts)

a) Assume drawing is done <u>without replacement</u>. Show that players B and C have equal chances of winning. What is the probability of winning for each of the three players?

A can win on draw 1, 4, 7,

$$\therefore P(A) = P(W_1) + P(W_4) + P(W_7)$$

$$= \frac{1}{7} + \frac{6}{7}\cdot\frac{5}{6}\cdot\frac{4}{5}\cdot\frac{1}{4} + \frac{6}{7}\cdot\frac{5}{6}\cdot \text{---} \cdot\frac{2}{3}\cdot\frac{1}{2}\cdot 1$$

$$= 3 \cdot \frac{1}{7} = \frac{3}{7}$$

Similarly $P(B) = P(W_2) + P(W_5) = \frac{1}{7} + \frac{1}{7} = \frac{2}{7}$

and $P(C) = P(W_3) + P(W_6) = \frac{1}{7} + \frac{1}{7} = \frac{2}{7}$

$$\therefore P(B) = P(C)$$

(This is like choosing 1 out of 7 positions for W. all are equiprobable)

b) Assume drawing is done <u>with replacement</u>.
What is the probability of winning for each of the three players?

$$P(A) = \frac{1}{7} + \left(\frac{6}{7}\right)^3 \cdot \frac{1}{7} + \left(\frac{6}{7}\right)^6 \cdot \frac{1}{7} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{1}{7} \cdot \left(\frac{6}{7}\right)^{3n} = \frac{\frac{1}{7}}{1 - \left(\frac{6}{7}\right)^3} = \frac{49}{127}$$

$$P(B) = \frac{6}{7} \cdot \frac{1}{7} + \left(\frac{6}{7}\right)^4 \frac{1}{7} + \left(\frac{6}{7}\right)^7 \cdot \frac{1}{7} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{1}{7} \left(\frac{6}{7}\right)^{n+1} = \sum_{n=0}^{\infty} \frac{1}{7} \cdot \frac{6}{7} \cdot \left(\frac{6^3}{7^3}\right)^n = \frac{\frac{1}{7} \cdot \frac{6}{7}}{1 - \left(\frac{6}{7}\right)^3} = 42/127$$

$$P(C) = 1 - (P(A) + P(B)) = \frac{36}{127}$$

c) Assume drawing is done <u>with replacement</u>. What is the probability that the winner is declared at the seventh trial (meaning that A wins on his third attempt)?

Geometric; $p = \frac{1}{7}$, $x = 7$
(special case)

$$P(x=7) = \left(\frac{6}{7}\right)^6 \cdot \frac{1}{7} = \frac{46{,}656}{823{,}543} = 0.05665$$

III. ***(4+4+4+4=16points)*** Two balls are drawn randomly and without replacement, from an urn containing two white and three black balls. Let X count the number of white balls in the selection.(Answer the following parts)

a) Show that the p.m.f. of X may be given by:

$$f(x) = \frac{6}{5(x!)[(x+1)!][(2-x)!]^2};\ x = 0,1,2.$$

Hypergeometric; $N_1 = 2$, $N_2 = 3$, $N = 5$, $n = 2$, $0 \le x \le 2$

$$f(x) = \frac{\binom{2}{x}\binom{3}{2-x}}{\binom{5}{2}} = \frac{\frac{2!}{x!\,(2-x)!} \cdot \frac{3!}{(2-x)!\,(x+1)!}}{\frac{5!}{2!\,3!}}$$

$$= \frac{\frac{(2!\,3!)^2}{5!}}{x!\,[(2-x)!]^2 (x+1)!} = \frac{6}{5\,x!\,(x+1)!\,[(2-x)!]^2}$$

b) **Complete the three missing table entries:**

x	0	1	2
$f(x)$	0.3	0.6	0.1

c)**Find the mean of X.**

$$\mu = E(x) = \sum_{x=0}^{2} x f(x) = 0 \cdot 0.3 + 1 \cdot 0.6 + 2 \cdot 0.1 = 0.8$$

d) **Calculate the variance of the distribution.**

$$E(x^2) = \sum_{x=0}^{3} x^2 f(x) = 0^2 \cdot 0.3 + 1^2 \cdot 0.6 + 2^2 \cdot 0.1 = 1$$

$$\sigma^2 = E(x^2) - \mu^2 = 0.36$$